FILOSOFIA DELLA SCIENZA

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Alla base di questa collana vi sono due idee guida. La prima è che i confini tra le discipline sussistano soprattutto per il piacere (e l'esigenza) di varcarli e che questa istanza sia più forte di qualsiasi implacabile "polizia di frontiera", tesa a impedire la libera interazione e lo scambio dialogico tra i diversi campi del sapere. Valeva ieri per la teoria di Copernico e per quella di Darwin, vale, a maggior ragione, oggi per le frontiere della cosmologia o per quelle della biologia e della fisica, per non parlare dell'informatica o dell'alta tecnologia.
La seconda idea è che la filosofia più interessante, come amava ripetere Ludovico Geymonat, è quella che si annida nelle pieghe della scienza, per cui è a quest'ultima, nelle sue diverse articolazioni e nei suoi svariati indirizzi, che vanno al di là di ogni artificiosa barriera tra "scienze della natura" e "scienze umane", che bisogna guardare per dare una risposta seria e credibile ad alcune delle grandi domande che la filosofia si è posta nel corso del suo sviluppo storico.
In questo quadro generale i singoli contributi che vengono proposti sono tutti contrassegnati da frequenti segni d'interpunzione metaforici, per stimolare quel tipo di lettura di cui parla Wittgenstein nei suoi *Pensieri diversi*: «Con i miei numerosi segni d'interpunzione io vorrei rallentare il ritmo della lettura. Perché vorrei *essere letto lentamente*». Non sono libri "usa e getta", da affrontare in maniera fugace e sbrigativa. Sono opere che esigono di essere lette seguendo e facendo propria la bellissima (e sempre attuale) massima attribuita a Svetonio, che è un richiamo all'importanza della meditazione: «*Festina lente*».

Daniele Chiffi

Kurt Gödel. Philosophical Explorations

History and Theory

Preface by Arcangelo Rossi

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00173 Roma
(06) 93781065

ISBN 978–88–548–4506–0

I[st] edition: January 2012

Alla mia famiglia

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Preface

This volume is dedicated to Kurt Gödel's work as a whole on the base of its deep unity and internal cross-fertilization. If these features were disregarded, even the best known of Gödel's results, i.e. his theorems on the incompleteness of elementary arithmetic, could not be fully understood, as they are strictly interconnected with Gödel's entire research program, notably the axiomatic method and constructivism.

In fact, as the author stresses, such results are not mere technical contributions, but also the consequence of Gödel's general approach to logic, foundations of mathematics, epistemology and even to the methodology of proof, key point of epistemology in general. They are even expressions of the fundamental philosophical distinction between intuition and symbol, extended from science to metaphysics as was illustrated by G. W. Leibniz at the base of his general conception of knowledge and of language in all the range of its functions to the point of characterizing Gödel's constructivism as much more realistic ("Platonist") than conventional intuitionism. Gödel even arrived, at least partially, at translating intuitionist into classical mathematics and *vice versa*, so showing the interconnections between the classical realistic approach and the constructive one. In fact, with all its computational content, and so going well beyond classical indirect demonstrations making appeal to *ab absurdo* and indirect proofs, Gödel's was, at the same time, a much more constructive and, though undoubtedly moderate, intuitionist approach. So, in the last part of his life, Gödel even adhered, as the author largely illustrates, to the phenomenological view of objectification of intuition and intentionality as active factors of knowledge, anyway irreducible to mere subjective psychological processes but, at the same time, also irreducible to mere mathematical and formal logical structures devoid of independent ideal content.

But even that was not enough. As the author clearly shows, Gödel did not limit himself to compose constructive and even intuitionist, though moderate, arguments with realistic (Platonist) ones on mathematics and logic. The author underlines that Gödel developed his foundational realistic approach also in frankly metaphysical and even mystical terms (as, in my opinion, in his attempted demonstration of the existence of God, also derived from Leibniz), not only in rational, causal and deterministic terms, even severely criticizing Kant's supposedly both subjectivist and phenomenist approach in favour of an ontological and noumenic one. Anyway, his both realistic and moderately constructivist viewpoint had an ontological and even theological foundation, quite opposite to B. Russell's "lay" foundation of logical moderate realism, which anyway converged with Gödel's predicativism and conceptualism through an axiomatic and at least partially constructive (i.e. predicativist as J. H. Poincaré's was) conceptualistic approach.

Anyway, Frege and Hilbert started a controversy on the foundations of logic and mathematics. Frege followed Kant's approach because of the necessity of integrating logical non-contradiction or formal coherence with logical and mathematical content in order to reach logical and mathematical truth. Hilbert, on the contrary, simply identified the logical and mathematical truth with the mutual coherence of formal finite signs lacking any sense. In fact, Gödel considered Kant's view too subjectivist but, like Kant, accepted intuitive content as a necessary integration of formal logic and mathematics. Gödel points out that the mathematical content is not identifiable (as suggested by the phenomenological approach) with mere subjective perceptions or intuitions translatable into symbols in a constructive calculus. For Gödel, Hilbert was anyway right for what regards the logical formal coherence to be maintained in his formalist and finitist approach, but he was wrong in disregarding the intuitive and constructive content of logic and mathematics as irreducible to the mere formal coherence of pure finite symbols.

The controversy between Frege and Hilbert, that is between the logicist and the formalist perspectives in mathematics, influences Gö-

Chapter I

The Frege-Hilbert Controversy

1.1. Kant's Views on Mathematics

The present status of logical notions like "axiom", "definition", and meta-logical notions such as completeness and consistency trace back to the development of the non-Euclidean geometries. I will analyse the Frege-Hilbert dispute in order to show a very important step in the genealogy of these concepts, which are embedded into two different points of view about the role of the new axiomatic method. In the present section I show the importance of the Kantian views on mathematics and their role in the foundational schools. In section 1.2., I sketch out some of the points of Hilbert's new axiomatic method, in 1.3. I will make a thorough analysis of Frege's objections towards Hilbert. In 1.4. I will point out Frege's semantic proof of logical independence *versus* Hilbert's arguments of independence. In 1.5., I will stress the differences between different types of definitions in mathematics, showing the development of some results of Frege and Padoa in the work of Beth. And lastly in section 1.6., I will present some concluding remarks about Frege's and Hilbert's view on logic, which I call the "philosophical view" and the "mathematical view".

The Kantian account of mathematics comes forth as a very important part in the developments of mathematics, especially in the foundational schools. I briefly introduce some Kantian themes in mathematics which will turn out to be fundamental later on.

According to Kant, the intuition and the construction of a concept distinguish mathematical judgments from metaphysical judgments.

Namely, mathematics deals with constructible concepts, while philosophy handles concepts that are already given, *viz*., the mathematical concepts are *constituted* and *constructed* by the definition, while in philosophy we can only *explain* our concepts. Hence, according to Kant, a *mathematical* definition is only a *constitutive* definition. This fact will also take place in the modern mathematical constructivism, as we will take notice further on in the present work. Kant observes that

> "Philosophical knowledge is the knowledge gained by reason from concepts; mathematical knowledge is the knowledge gained by reason from the construction of concepts. To construct a concept means to exhibit a priori the intuition which corresponds to the concept. For the construction of a concept we therefore need a non-empirical intuition. The latter must, as intuition, be a single object, and yet none the less, as the construction of a concept (a universal representation), it must in its representation express universal validity for all possible intuitions which fall under the same concept" (KANT 1965, B 741)[1].

Examples of the constructions of mathematical concepts are the following:

> "the determination of an intuition a - priori in space (figure), the division of time (duration), or even just the knowledge of the universal element in the synthesis of one and the same thing in time and space, and the magnitude of an intuition that is thereby generated (number), -- all this is the work of reason through construction of concepts, and is called mathematical" (KANT 1965, B 752).

Notice that space and time are strictly connected with the idea of the construction of a mathematical concept. But this fact does not imply that the mathematical objects are full-blooded temporal (and spatial) objects, since the intuition of time only takes place through the general doctrine of motion (KANT 1965, B49)[2]. The Kantian account

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[1] In the preface of KANT (1970), we also find expressed a similar idea. For an early analysis of Kant's views on mathematics in the spirit of Logicism, see (COUTURAT 1904). A Neokantian reply to Couturat is (CASSIRER 1907).

[2] See FRIEDMAN (1992, chapter 2).